From Assyriology to Renaissance Art

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Knowing the repulsive force which any kind of numerical data exercises on humanists I did not dare to give this paper its appropriate title "From 29;31,50,8,20^d to 29^d 12^h 793^p". However, these two numbers could reveal to any attentive reader the subject matter I intend to discuss here. Let us begin with the second number which deals with 29 days and 12^h plus a little tail and compare it with the first one 29;31,... which, in sexagesimal notation, means 29^d 31/60 ..., i.e. again 29 30/60 = 29½ days plus a tail of smaller units. Obviously we are here concerned with "lunar months," i.e. with time intervals of about 29½^d which in calendrical usage must lead to "months" alternating between 29 and 30 days. This is historically important topic since sequences of "hollow" (29^d) and "full" (30^d) months characterize the Islamic calendar and were of fundamental importance in the Mesopotamian civilization, in the Jewish calendar and hence in the Easter "computus" and in various other contexts throughout history.

Having identified, at least in outline, the direction of our inquiry, we can now begin at the beginning which, of course (following Sam Kramer), lies in Sumer. Indeed there existed in the third millenium B.C. in Mesopotamia a small unit of length, called "barleycorn" (še) representing a fraction (1/6) of a finger-breadth. The finger-breadth in turn is a fraction of the palm, the palm of the cubit, such that 1 cubit = 3,0 = 180še. We shall not follow here the intricacies of the history of Mesopotamian metrology[1] but only point to a well-known phenomenon: measures can lose their specific meaning and become terms for fractional parts in general: the Roman *as*, e.g., originally a weight, becomes a term for 1/12. Similarly, the barleycorn, embedded in a sequence of sexagesimally arranged units, retains only its fractional significance as 18 units of 60ths, i.e., as 1/1080. In this form we find it in Babylonian astronomy of the Seleucid-Parthian period and then in Hebrew time-reckoning as halakim ("parts"), representing 1/1080 of one hour. In our second number the 793^p mean just these parts. And, making use of the relations 1080^p = 18,0^p = 1^h and 1^d = 24^h, one finds that

[1] For the "Barleycorn" cf. ACT I p. 39, Neugebauer [1937] p. 280, [1945] p. 12f., Sachs [1947] p. 70f., Maimonides, *Sanctification*, p. 117.

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$12^h\ 793^p$ are exactly $0;31,50,8,20^d$. Hence the two numbers mentioned at the beginning represent the same interval of time, a little more than 29½ days (in modern units $29^d\ 12;44,3,20^h$, i.e. 44 minutes 3 1/3 seconds beyond $29½^d$).

Hence we know now that we are dealing actually only with one single parameter for the length of a lunar month. Its value, a little more than $29½^d$, leads to an important consequence: a real lunar calendar based on such months cannot consist of simply alternating full and hollow months. The history of the Semitic calendars is a vivid illustration of this fact.

Before going any further, let us answer one question which one will naturally ask: how could one come to an interval of time of such accuracy that it involves a fraction (1/3) of a second? We shall not attempt to give a detailed answer, but it should be made clear that observation with accurate instruments can be securely excluded. Let us note that twelve months of accurately $29½^d$ length would be eleven days shorter than a "year" of 365^d. Counting these deficits during a few years allows us with every step to obtain a sharper average value for the length of twelve real lunar months. Hence simple counting over a relatively small number of years would lead to such an average (as we know now nineteen years or its multiples would give comparatively stable estimates). In recognition of this origin from averages we shall call our parameter the length of the"mean synodic month."

The Seleucid Period: ACT

No more drastic discontinuity in the history of ancient astronomy can be imagined than the creation of mathematical astronomy in the Babylonian ephemerides and procedure texts (for which the acronym ACT has become widely used) in the Seleucid period. If astronomical phenomena had been considered since the earliest Mesopotamian period as celestial omina (or, in later periods, indicative of astrological facts) the authors of the ACT material ("Scribes" from the temples of Babylon and Uruk) dropped all these traditional connections and analyzed lunar and planetary motion in a strictly mathematical fashion comparable only to the approach of Hipparchus and Ptolemy.

That the men who created this new science were fully aware of the revolutionary character of their approach cannot be doubted. Their profession may well have originated in a practical context, the prediction of the dates of the first visibility of the lunar crescent that regulated the civil calendar. The discovery of the nineteen-year cycle[2] (around 600 B.C.) was one result of these efforts. But the methodology displayed in the ephemerides of the ACT scholars left the calendrical level far behind. Here an entirely new approach makes its first appearance: a systematic mathematical analysis of the lunar and solar motion and of the planetary

[2] For the history of this important cycle, cf. Bowen-Goldstein, *Meton*.

phases producing results on which later Greek kinematic interpretation could be built. The origin of mathematical astronomy in Babylon and Uruk can thus be dated to the years around 400 B.C.

If this chronology is securely established, there remains a strange puzzle in the character of the sources available in abundance among the tens of thousands of contemporary records. Texts which are known as "astronomical diaries" (and related groups)[3] contain reports on observations that involve fixed stars, e.g., the report about a planet being observed on a certain day a number of cubits "above" or "below" (or "before" or "behind") a certain bright star. Why were such observations consistently made and recorded? They are never mentioned in the ACT material and are, in fact, useless for the composition of the mathematical texts. And conversely: a reference to a phenomenon determined in a contemporary ACT ephemeris or procedure text never appears in a "Diary."

I see no other way of explaining this fact than as evidence of a profound split among the professional "astronomers" of the Late Babylonian period. For the authors of the ACT the fixed stars were of no interest whatsoever. The men of the "Diaries," in contrast, followed the time-honored way of providing the data for celestial omina that in the last analysis concern events beyond the power of arithmetical rules. Nothing compels us to assume that these two groups of professional men considered one another with particularly kind feelings. Even among the authors of the ACT one might suspect a spirit of competition between the schools of Babylon and of Uruk, perhaps reflected in the contemporary "systems" which we now call A and B, the latter being well attested in Greek context.

Any attempt to give here even the most general summary of the contents and structure of the ACT material is out of the question. Fortunately, it suffices for our topic to stress a few generalities: First, Fig. 1 illustrates the dates and coverage of the extant texts, showing also the division into "System A" and "System B." Particularly, the lunar ephemerides, requiring many interrelated numerical columns, are very carefully written (cf., e.g., Fig. 2). Many of these columns (cf. Fig. 3) are based on sequences of increasing and decreasing constant differences which in graphical (modern!) representation would appear as zigzag graphs, varying between fixed extrema M and m (Fig. 4). Their arithmetical mean $\frac{1}{2}(M + m)$ is the "mean value" of the quantity under consideration.

As an example of such a column we quote here only one, representing the variable length of the synodic month by adding or subtracting $d = 22{,}30{,}0$ but never transgressing $M = 4{,}29,\ 27{,}5$ or $m = 1{,}52{,}34{,}35$. This gives for the mean value 3,11,0,50, the first digit counting "large hours" (H) such that $6^{H} = 1^{d}$ or $1^{H} = 4^{h} = 0;10^{d}$. Hence the above mean value

[3] Now in the process of publication; cf. Sachs-Hunger, *Diaries*.

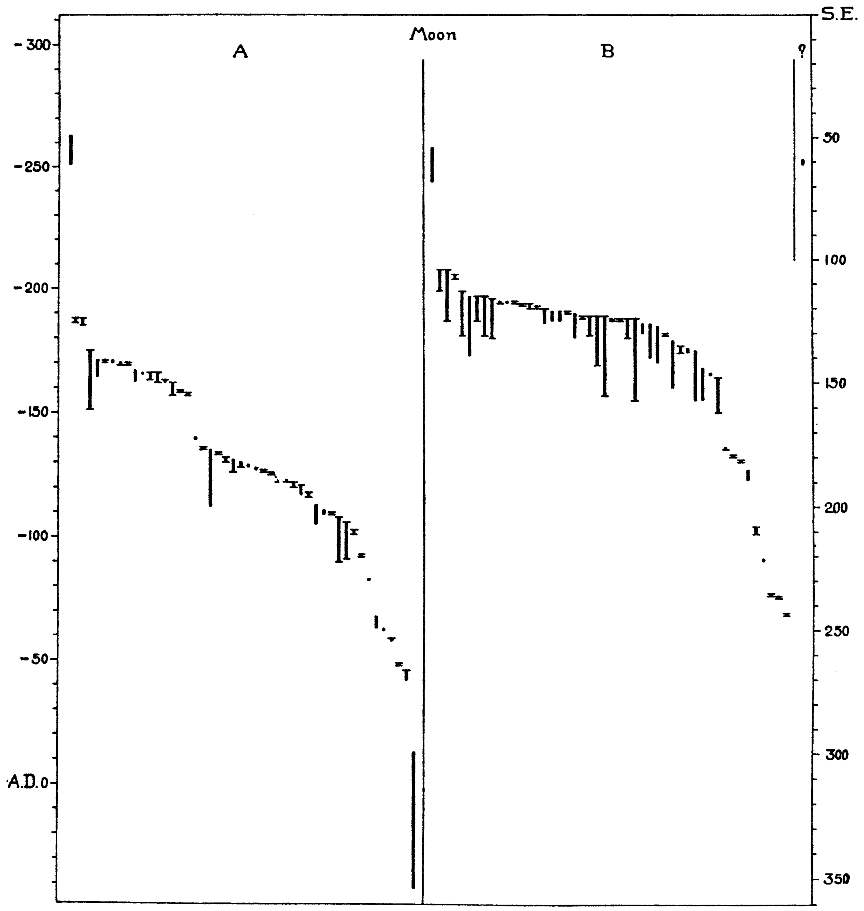


FIGURE 1

expresses the fraction 0;31,50,8,20^d, i.e. the excess over 29^d of the mean synodic month as shown in the first version of our famous parameter (cf. p. 391).

FROM BABYLON TO ROMAN EGYPT

Exactly one hundred years ago appeared a little Book, *Astronomisches aus Babylon*, written by the Jesuit Father J. Epping in cooperation with the Assyriologist Father J. N. Strassmaier, which contained a whole series of extraordinary discoveries: for the first time the words "Babylonian astronomy" became endowed with a concrete meaning, fully comparable to that of "Greek astronomy" enshrined in the *Almagest* or the *Handy Tables*.

After Epping's death in 1894 his work was continued (again with the undefatigable help of P. Strassmaier) by F. X. Kugler, S. J., whose first large work, *Die Babylonische Mondrechnung* (1900), is of special interest in

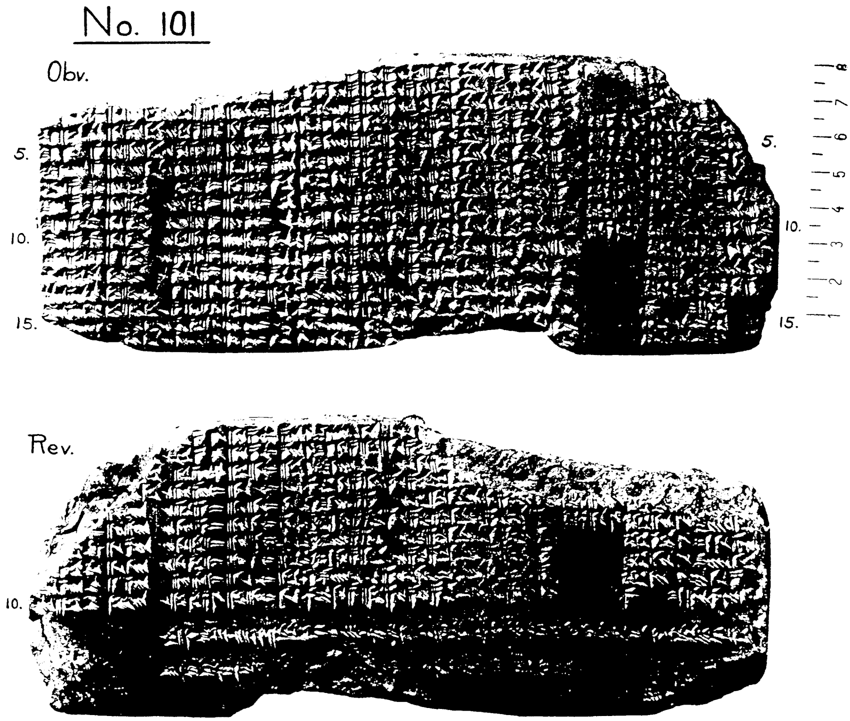


FIGURE 2

the present context. Here Kugler succeeded in showing that in a group of Babylonian lunar ephemerides from the hellenistic period were embedded parameters which about one century later were known to Hipparchus, as Ptolemy reported in the *Almagest*.[4] And among these we find again our parameter $29;31,50,8,20^{d}$ for the mean synodic month.

But Kugler in his analysis of lunar ephemerides discovered much more than the mere knowledge of a parameter which could have been derived from calendrical cycles. He showed that one column of variable numbers (he called it col. G) had our parameters only as mean value, whereas each of the other numbers represented the variable duration of the lunar month, following a strictly periodic pattern. This, however, had the further implication that this list of numbers was computed without allowing for a variable solar velocity. Kugler then succeeded in showing that additional columns contained corrections to G such that the combined solar and lunar angular velocities were accounting for the complex variations in the actual length of the synodic month. Hence he suddenly became aware of the fact that the complicated machinery of Ptolemaic epicycles and eccentricities were developed on the basis of an equally sophisticated set of numerical procedures extant in Babylon at

[4] CF. Kugfler, Mondr. p. 109ff.

No. 121

Obv.	[T1]	[0]	I	II	III	IV	V	Obv.
1.	XII₂	45,55,48,53,20	[1,29,31,1]3,20	1,56,41,40	11,40,10	4,21,31,40	14,2[2,30]	1.
	3,1 I	45,22,28,53,20	[44,8]44,26,40	2,2,41,40	12,16,10	3,59,1,40	7,3[5]	
	II	44,49,8,53,20	[40]24,26,40	2,8,41,40	12,52,10	3,36,31,40	4[7,30]	
	III	45,18,2,13,20	[9,5]8,26,40	2,14,41,40	13,28,10	3,14,1,40	6	
5.	IV	45,51,22,13,20	[55]49,48,53,20	2,20,41,40	14,4,10	[2,]51,31,40	12,4[7,30]	5.
	V	46,24,42,13,20	[1,4]2,14,31,6,40	2,26,41,40	14,40,10	2,29,1,40	19,3[5]	
	VI	46,58,2,13,20	[1,2]7,41,26,40	2,32,40	15,16	2,6,31,40	15,3[7,30]	
	VII	47,31,22,13,20	[40]19,4,26,40	2,26,40	14,40	2,1,7,30	8,5[0]	
	VIII	48,4,42,13,20	[7,]54,37,46,40	2,20,40	14,4	[2,]23,37,30	2,[2,30]	
10.	IX	47,48,6,40	[19]42,44,26,40	2,14,40	13,28	2,[4]6,[7,3]0	[4,45]	10.
	X	47,14,46,40	[1,6,]57,31,6,40	2,8,40	12,52	3,8,3[7,30]	[11,32,30]	
	XI	46,41,26,40	[1,53]38,57,46,40	2,2,40	12,16	3,31,[7,30]	[18,20]	
	XII	46,8,6,40	[1,1]7,6,55,33,20	1,56,40	11,40	3,53,[37,30]	[16,52,30]	

Rev.	[T2]	I	II	III	IV	G V	Rev.
1.	3,1 I	[45,42,53]20	1,4,57,36,40	2,2[9,50]	[14,59]	[2,11,45]	1.
	II	[45,9,33]20	19,48,3,20	2,2[3,50]	[14,23]	[2,34,15]	
	III	[44,57,37]46,40	25,9,34,26,40	2,1[7,50]	[13,47]	[2,56,45]	
	IV	[45,30,57,4]6,40	34,40,32,13,20	2,1[1,50]	[13,11]	[3,19,15]	
5.	V	[46,4,17,]46,40	1,20,44,50	2,5,[50]	[12,35]	[3,41,45]	5.
	VI	[46,37,3]7,46,40	1,49,31,32,13,20	1,5[9,5]0	1[1,59]	[4,4,15]	
	VII	[47,10,57,]46,40	1,2,20,34,26,40	1,53,50	11,23	[4,26,45]	
	VIII	[47,44,17,]46,40	14,36,16,40	[1,]53,51,40	11,23,1[0]	[4,9,39,10]	
	IX	[48,8,3]1,6,40	33,32,14,26,40	1,59,51,40	11,59,10	[4,47,9,10]	
10.	X	[47,35,1]1,6,40	45,7,25,33,20	2,5,51,40	12,35,10	[3,24,39,10]	10.
	XI	[47,1,5]1,6,40	1,32,9,16,40	2,11,51,40	13,11,10	3,[2,9,10]	
	XII	[46,28,3]1,6,40	1,38,16,12,13,20	2,17,51,40	13,47,10	[2,39,39,10]	
	3,2 I	[45,55,11,]6,40	52,21,1,6,40	2,23,51,40	14,23,10	2,[17,9,10]	

FIGURE 3

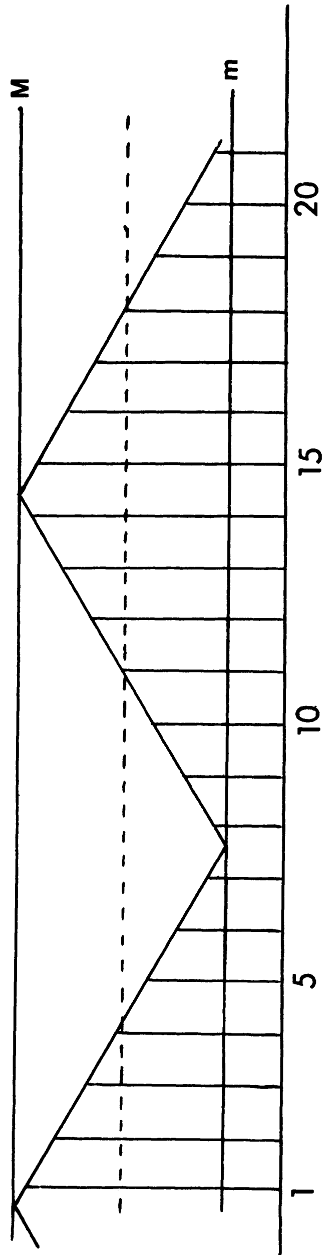
least since about 250 B.C. Few historical studies had such far-reaching consequences for our insight into ancient cultural developments as did the works of Epping and Kugler.[5]

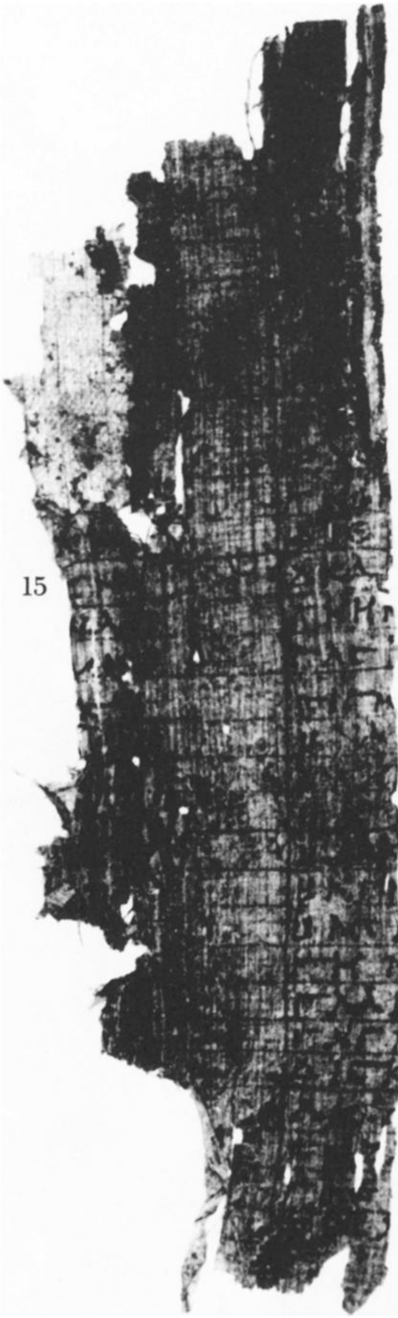
Thanks to the *Almagest* we have at least some information about the systematic progress made by Ptolemy since Hipparchus (around 150 B.C.). But the question remains: how and in what form did Hipparchus or Ptolemy receive data belonging to Babylonian mathematical astronomy? By sheer accident a new aspect of this problem has recently been uncovered. A small papyrus fragment containing a slightly damaged column of Greek numerals was communicated to me (cf. Fig. 5), after about 25 years in undisturbed private existence.[6] Obviously we had here a linear zigzag function with constant difference 22,30, reflected at $M =$ 4,29,27,5 and $m =$ 1,52,34,35, the parameters listed above (p. 393) of Column G of a lunar ephemeris whose mean value corresponds to 29;31,50,8,20^d for the length of the mean synodic month.

Of course, we know nothing about the date and provenance of this fragment. That it was written during Roman times in Egypt seems fairly

[5] Neither Epping nor Kugler are mentioned in the DSB (while Sarton received 7 pages). For the relevant biographical data, cf. the *New Catholic Encyclopedia* (1967).

[6] Cf. Neugebauer [1989].


M
m
1
5
10
15
20



[αφ]	4,2[2, 34, 10
	4, 0, 4,[10
	3,3[7, 34, 10
	3,1[5, 4, 10
	2,5[2, 34, 10
	2, 30, [4, 10
	2, 7,[34, 10
πρ	2, 0, [5
	2, 22,[35
	2, 45, [5
	3, 7,3[5
	3, 30, [5
	3, 52,3[5
	4, 15, [5
αφ	4, 21,[19, 10
	3, 58,[49, 10
	3, 36,1[9, 10
	3, 13,4[9, 10
	2, 51,[19, 10
	2, 28,[49, 10
	2, 6,[19, 10
πρ	2, 1, 20
	2, 23,5[0
	2, 46,[20
	3, 8,5[0
	3, 31,2[0
	3, 53,5[0
	4, 16, 20
[αφ]	4, 20, [4, 10
	3, 57,3[4, 10
	3, 35, [4, 10
	3, 12,3[0, 10

FIGURE 5

certain and is supported by a sign for the number 0 (in line 8) which is also attested in papyri from the second or third century A.D. As to the provenance one may safely eliminate Alexandria since humidity and continued human occupation practically excludes survival of papyrus fragments.

Hence we must now recognize that about the time of Ptolemy or later, someone in Egypt had access to a Babylonian ephemeris. This must be understood in a much wider sense than knowledge of the Babylonian value for the mean synodic month in calendrical context. A complete column G covering more than two years has meaning only within accurate lunar theory based on variable lunar and solar motion. Hence our fragment demonstrates the existence of persons, not known to us from contemporary treatises, who were studying Babylonian astronomy (e.g., intelligent professional astrologers), from ephemerides written in Greek, thus without the need to consult cuneiform tablets. Needless to say, this opens an entirely new aspect on the transmission of Babylonian astronomy to the Greeks and on the spread of scientific knowledge in late antiquity.

The Halakim

Why the Jews divided the hour in 1080 parts (sexagesimally 18,0) we do not know excepting the unquestionable Babylonian derivation from the "Barleycorn." Also the larger unit of the "hour," one twenty-fourth of the day, is based on the Babylonian sexagesimal division of the circle in degrees (uš), 60 of which (or 4^h) are a fundamental unit of time in Babylonian mathematical astronomy (cf. above p. 393).

The first evidence for the Jewish version of the Babylonian parameter which (in "System B") represents the length of the mean synodic month appears in the Babylonian Talmud at the end of the first century A.D.,[7] ascribed to R. Gamaliel. Another early occurrence is in the Megillah.[8] Biruni in his *Chronology* (written A.D. 1000) gives a detailed description of the Jewish calendar.[9] And he remarks that the Jews themselves say that they received the value for the mean synodic month from the Babylonians.[10]

Not quite two centuries after Biruni we can again follow in detail the arithmetical procedures in the Jewish calendar in a treatise on *The Sanctification of the New Moon*, written in Cairo by the famous scholar Maimonides (Saladin's physician). In the subsequent chapters, however,

[7] Hebrew-Engl. edition, p. 25[a]. I owe this reference to Professor A. Wasserstein.
[8] Cf. Ideler I p. 542, but I could not identify the passage.
[9] *Chron.* ch. VII (transl. Sachau p. 141); also p. 64. For the "Barleycorn" (1/6 fingerbreadth) in Syriac and Arabic context, cf. e.g. F. Nau, *Le Livre de l'Ascension de l'Esprit* (Paris 1899) 178 [202].
[10] *Chron.* 65.

where Maimonides discusses lunar theory more or less following Ptolemaic methods, no halakim appear. One may perhaps say in general that their use was restricted to purely calendrical matters.

Books of Hours

Historians of Renaissance Art rightly pay much attention to the splendidly illuminated manuscripts known as Books of Hours, very popular in France and Italy in the fourteenth and fifteenth centuries. These books also contain, usually in the beginning, pages depicting the monthly labors and calendrical matters and occasionally special sections at the end may concern even technical matters related to Easter dates. There "Sunday Letters" and "Golden Numbers" make their appearance, usually to the dismay of historians of art.

Only recently, however, Dr. Edith Kirsch brought into the discussion such a calendrical appendix, from a manuscript in Modena (Lat. 842), which sheds unexpected light on the astronomical background of the calendrical basis of late Medieval and Renaissance Easter-computus. There we find on sixteen pages what we shall euphemistically call "tables" (cf. Fig. 6) for new moons concerning the years 1375 to 1393. I shall not go into any discussion of the structure and meaning of these tables, but turn directly to our topic, the "mean synodic month."

On the first page (concerning Jannuarius) we read in the fourth line A B 19 676. The B we can ignore since it is only the current number in the alphabet from A to F representing the days of the week (the "Sunday Letters"). But the letters in the first column are of a non-trivial order. Closer inspection shows that all nineteen letters of the alphabet from A to T are used, obviously counting the years in a nineteen year cycle and thus honored with the name "Golden Numbers."

Knowing now that the golden numbers are nothing but cycle numbers, we follow for a certain year, e.g., F = 6, the numbers found in consecutive months:

Jan	7	17	160
Feb	6	5	953
Mar	7	18	666

These numbers have constant differences, assuming that the last one cannot transgress 1080, while the one before is limited by 24^h. Indeed:

$$160 + 793 = 953 \qquad 953 + 793 = 1746 = 1080 + 666.$$

The days increase by 29 from month to month and hence we have again reached the canonical value for the difference: $29^d\ 12^h\ 793^p$.

It would have been natural to use the year A = 1 for an example of constant difference but already the second month is affected by a scribal error. Mistakes abound in the whole table; even in our example from year F, an error of -2^p is embedded. But errors are occasionally an important tool to establish the structure of a text. Our computist was seriously disturbed in the seventh year where two mean conjunctions occurred in the same calendar month, at the beginning and at the end of

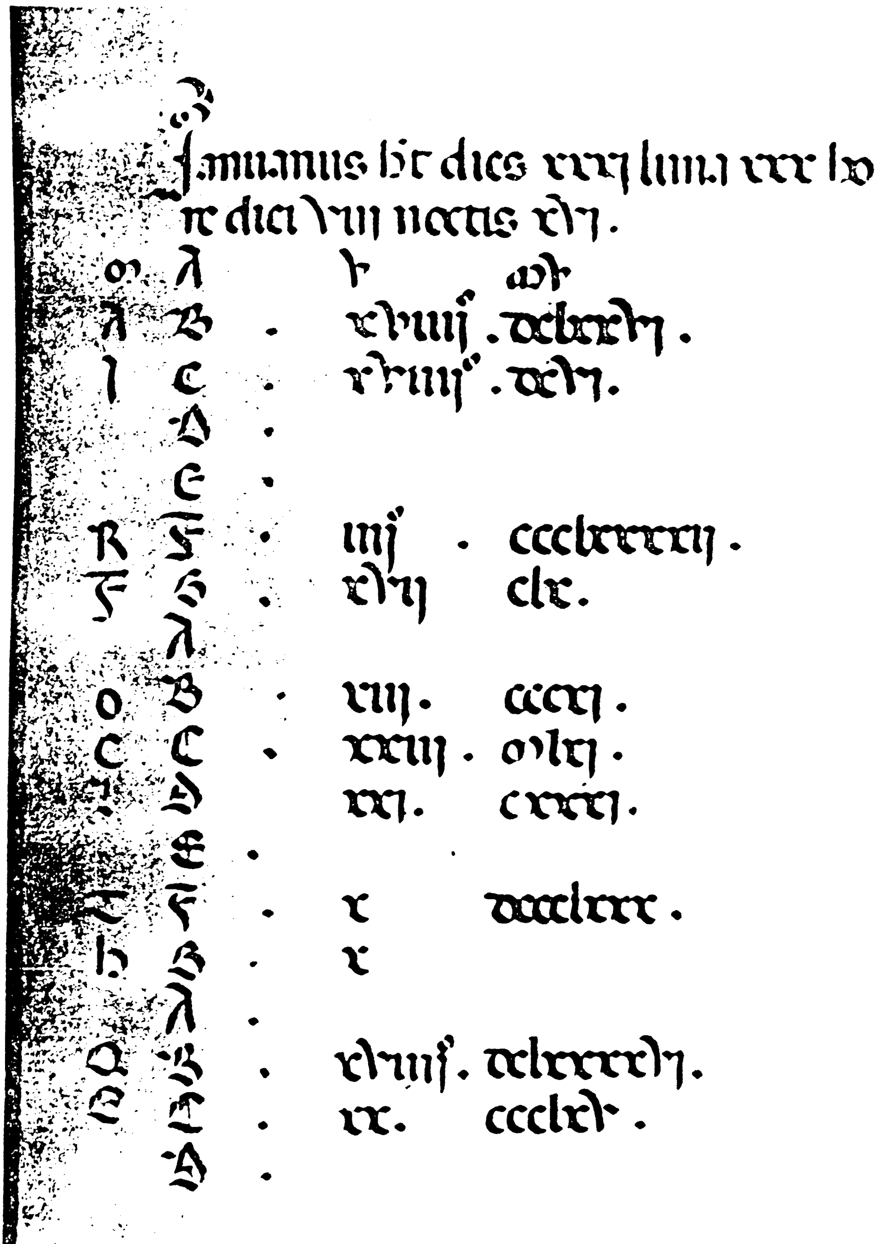


FIGURE 6

August. This caused him to skip a whole month, not to mention smaller numerical errors. At this point he must have realized that something was wrong and he turned to a desperate way of correction: he assumed for year T (= 19) a proper last date (in December) and restored the earlier dates by computing in reverse order.

It is not surprising to find that this sequence of data is also vitiated by many computing errors. Of some interest, however, is the discovery of how he chose the last value in year T: Dec 4 6^h 963^p. Computing from it the first value of year A of the next cycle, one finds Jan 2 19^h 676^p, i.e., exactly the same value from which we started in our cycle:

$$\text{Dec } 4\ 6^h\ 963^p + 29^d\ 12^h\ 793^p = \text{Jan } 2\ 19^h\ 676^p.$$

Obviously our computist postulated strict periodicity after nineteen years. This, however, is incorrect since the 19-year cycle concerns (tropical) solar years, not calendrical julian years, with a discrepancy of about ¼ day in each cycle. Indeed, correct computation starting with Jan 2 19^h 676^p would have resulted for the next cycle in Jan 2 12^h 191^p, hence with a deficit of about 7^h.

But of greater interest than the determination of a theoretical misunderstanding is the insight in the basic structure of the calendrical data in a Book of Hours. We know now that a highly accurate value for the length of the mean synodic month underlies its dates. This constant difference and one initial date (close to some true conjunction) completely determines the sequence of full and hollow months as well as those calendar years which contain thirteen lunar months (seven such "intercalary" years occur in each cycle). The study of the famous Books of Hours of the Duke of Berry (about 1415) seemed to show that the sequence of full and hollow months (within the astronomically permissible limits) was chosen arbitrarily. It is now clear that the knowledge of our exact parameter was still alive around 1400 and could (or should) have been used for the exact determination of the fundamental data of the lunar calendar.

Summary

In retrospect we must stress the extremely fragmentary character of our information. Nevertheless it cannot be doubted that an important parameter of Babylonian lunar theory remained in perpetual use into the calendrical structure of Renaissance works of art. We can also be certain that its astronomical significance was still fully understood in Roman Egypt, even outside the schools of professional astronomers like Ptolemy or Theon and we know that a Hebrew version existed during the same period but probably reduced to simple calendrical usage. We know that it survived in this form into the late Middle Ages in Muslim Egypt but we cannot say how this version reached the computists of the Renaissance.

Our inquiry has stretched over at least sixteen centuries. If we compare the two endpoints of our voyage, Fig. 1 on p. 394 and Fig. 6 on

p. 401, we can only note a dramatic decline from high competence and accuracy to simple incompetence in the handling of a purely arithmetical pattern. But history is a very complex phenomenon: only two centuries later, Kepler and Galileo initiated the "Astronomia Nova" which left far behind all the ingenious procedures developed in Hellenistic Babylon or Roman Egypt.

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